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Quantum tunneling in low-dimensional semiconductors mediated by virtual photons

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ABSTRACT

Quantum tunneling, a phenomenon that has no counterpart in classical physics, is the quantum-mechanical process by which a microscopic particle can transition through a potential barrier even when the energy of the incident particle is lower than the height of the potential barrier. In this work, a mechanism based on electron/hole annihilation and creation with the participation of virtual photons is proposed as an alternative to explain quantum tunneling processes in semiconductors. Finally, tunneling times are discussed within the proposed framework.

I. INTRODUCTION

Tunneling is a purely quantum-mechanical process by which a microscopic particle can penetrate a potential barrier even when the energy of the incident particle is lower than the height of the barrier.[1] In classical mechanics, a particle with energy E, which encounters a potential barrier V_0 on its path will reflect from it if $V_0 > E$. However, the quantum-mechanical description allows for the particle to be transmitted through the potential barrier. Nevertheless, in addition to being a counterintuitive phenomenon, justifying that tunneling occurs even if the energy of the incoming particle is smaller than that of the barrier has traditionally posed a philosophical puzzle.

In the present work, quantum tunneling processes through potential barriers in semiconductors are interpreted within the framework of quantum electrodynamics (QED) making use of the concept of virtual photons, i.e., transient intermediate states of the electromagnetic field.[2] The proposed model circumvents the traditional paradox of a particle with energy lower than that of the potential barrier being able to tunnel through it. Furthermore, in relation to the tunneling time, the proposed mechanism is consistent with the Hartman effect and recent developments.

II. QUANTUM TUNNELING

Quantum tunneling can be considered a consequence of describing the physical state of a particle using the Schrödinger equation since the wavefunction is not required to be zero inside the barrier. Accordingly, there is a probability different from zero to find the particle into the classically forbidden region. Different methods are commonly employed to calculate the transmission (or reflection) probability, the WKB approximation being the most widely used.[3]

The commonly accepted expression for tunneling through a one-dimensional potential barrier of height V_0 and width a is given by[4]

$$T = \frac{1}{1 + \left(\frac{V_0^2}{4E(V_0-E)}\right)\sinh^2(\kappa a)}, \tag{1}$$

with E being the energy of the incident particle and $\kappa = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$.

In the limit case where $\kappa a \gg 1$, i.e., extremely large potential barrier height V_0, the following approximation is obtained:

$$T = \left(\frac{16E(V_0-E)}{V_0^2}\right)e^{-2\kappa a}. \tag{2}$$

From Eqs. (1) and (2), it follows that the transmission coefficient rapidly decreases with increasing barrier width, particle mass, and energy difference $(V_0 - E)$.

III. TUNNELING VIA VIRTUAL PHOTONS

In a similar way that electromagnetic waves can spread across classically forbidden regions in the form of evanescent waves, which in QED-based studies are identified with virtual photons,[5] a tunneling mechanism based on electron/hole annihilation and creation by virtual photons is proposed below to provide a physical significance to the quantum tunneling process. As a matter of fact, some rigorous mathematical analogies between classical optics and quantum mechanics have been identified.[6] One of the better known and widely exploited analogy concerns the solutions to the time-independent Schrödinger and time-independent Helmholtz equations.[7,8] Furthermore, tunneling particles, as well as evanescent modes, are not observable inside the barrier. As such, it is here hypothesized that a parallelism can be established between evanescent modes and tunneling particles so that both can be identified with virtual photons.

Accordingly, the quantum tunneling mechanism here proposed is schematically depicted in Fig. 1, particularized to an electron in a semiconductor. In the diagram shown in Fig. 1, portraying the Feynman diagram for the lowest-order term of the suggested mechanism, the quantum tunneling process is described as the successive individual processes in which an electron and a hole enter (electron/hole annihilation), virtual photons are exchanged through the potential barrier, and finally an electron and a hole emerge, i.e., electron/hole pair formation.

Accordingly, an electron coming from the left would annihilate with a hole, both particles disappearing at the left of the barrier, and through the mediation of a virtual photon, an electron/hole pair would be created at the right of the barrier. The model here proposed circumvents the "classical" paradox of a particle with energy lower than the barrier height being able to "surmount" the barrier. In this model, the annihilation of incoming particles and the generation of particle/antiparticle pairs on the other side of the barrier avoid the previously mentioned paradox.

The overall process here presented can be described by the amplitude M, which is the quantum-mechanical amplitude for the process to occur. Using the Feynman rules, the amplitude for the QED process would be given by the following expression:

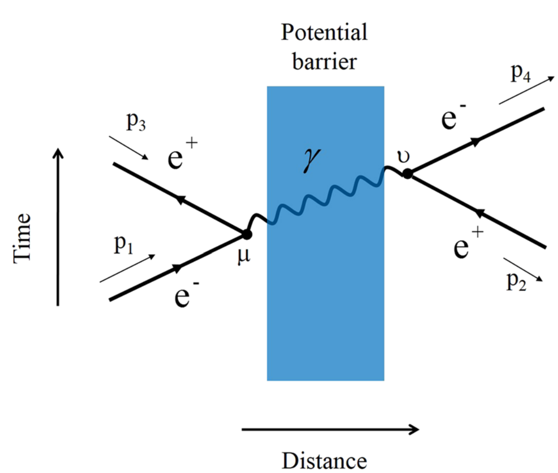

FIG. 1. Feynman diagram for the lowest-order term of the proposed mechanism for quantum tunneling through a potential barrier. At this order, the only possible intermediate state is a photon (γ).

$$-iM = \bar{\upsilon}(p_3)\left[ie\gamma^{\mu}\right]u(p_1)\frac{-ig_{\mu\upsilon}}{q^2}\bar{u}(p_4)\left[ie\gamma^{\upsilon}\right]\upsilon(p_2). \quad (3)$$

In the previous equation, $p_i = (E, \vec{p}_i)$, with p_1 and p_4 being the initial and final electron momenta, respectively, while p_3 and p_4 are the initial and final hole momenta, respectively. Accordingly, $q = p_1 - p_3 = p_4 - p_2$ so that $q^2 = (E - E)^2 - (\vec{p}_1 - \vec{p}_3)^2$ with $q^2 < 0$. u and $\bar{u}$ are spinors for incoming and outgoing electrons, while υ and $\bar{\upsilon}$ are spinors for outgoing and incoming holes, respectively. Finally, $\frac{-ig_{\mu\upsilon}}{q^2}$ is the photon propagator and γ^{μ}, γ^{υ} are 4×4 matrices which account for the spin-structure of the interaction. The spin-averaged matrix element would be given by $\left\langle |M_{fi}|^2 \right\rangle = \frac{1}{4}\sum_{spins}|M|^2$.

To calculate the total transmission coefficient, in addition to considering the amplitude M, the transmission amplitude of the photon, which might be virtual or real, needs to be taken into account. In the case of a photon, the transmission probability, T, will be related to its optical thickness, τ, by $T = e^{-\tau}$. The optical thickness is directly proportional to the attenuation coefficient and the thickness of the medium.

As such, the total probability amplitude will be obtained by multiplying the individual probability amplitudes of the single processes, i.e., electron/hole annihilation and creation (given by the spin-averaged matrix element) and transmission of the photon (real or virtual) through the barrier. The overall expression thus agrees with the observed dependence of the tunneling current with barrier thickness, following an exponentially decaying behavior [Eq. (2)].

Of course, the proposed model can be generalized to any particle since every particle has an associated antiparticle with the same mass but opposite charge (except photons), which is a consequence of the quantum field theory, given that particles and antiparticles are excitations of the same field. Particle-antiparticle pairs can annihilate each other, producing photons, which can be real or virtual. It is worth stressing that the proposed model does not preclude the participation of real photons. Since the charges of the particle and antiparticle are opposite, the total charge is conserved.[9] Regarding the generation of the required antiparticles and according to QED, quantum fluctuations, which are a consequence of Heisenberg's uncertainty principle, would be able to produce particle-antiparticle pairs. Particles remain virtual until promoted to real by conversion of energy via pair production.

IV. CONSIDERATIONS ON THE TUNNELING TIME

The time taken by a particle while tunneling has been the subject of long dispute (see, for instance, Refs. 10 and 11). Crucial to the tunneling-time problem is the fact that a semi-classical estimate of the velocity of a particle becomes imaginary since its kinetic energy inside the barrier is negative.[12] Accordingly, making the obvious approximation that the duration of a tunneling event is the barrier width divided by the velocity yields unphysical results. Many more sophisticated approaches have, therefore, been devised, although no satisfactory solution has been found so far. In fact, there are no well-constructed dynamical observables that could be used to determine tunneling times. It is worth mentioning that the interest in tunneling time has increased lately as a consequence of the development of experimental techniques which allow measuring events in the attosecond range (a current review can be found in Ref. 13). In this line, a recent study has put an upper limit of 1.8 as on any tunneling delay.[14] Nevertheless, although most experimental results seem to indicate that quantum tunneling is not an instantaneous process, there is no overall consensus from a theoretical point of view.[13]

In Fig. 1, the photon line has been deliberately drawn diagonal, given that the process, in principle, can proceed via both t-channel and s-channel photon exchange. As such, the mechanism proposed in this work is in accord with the Hartman effect[15] by which there is a finite time delay, although the delay time for a quantum tunneling particle is independent of the thickness of the potential barrier above a given value. More importantly, as pointed out by Hartman, this delay is shorter than the "equal" time, i.e., the time a particle of equal energy would take to transverse the same distance L in the absence of the barrier.[16] Accordingly, the participation of virtual (or even real) photons in the overall tunneling process would support this finding, given that light propagates faster than electrons (or any other massive particle).

Along this line, recent studies[17,18] have shown that the tunneling time vanishes and is independent, not only of the width but also of the height of the barrier for a square and a symmetric or asymmetric Eckart barrier potential, thus generalizing the Hartman effect to one-dimensional time-independent potentials. Additionally, it was also demonstrated that for a square barrier, a vanishing tunneling time does not lead to experimental measurement of speeds greater than c. In this regard, within the framework of quantum electrodynamics, instantaneous transitions are allowed for virtual particles, i.e., "space-like" transitions. This would be represented by a horizontal photon line in Fig. 1, which would be consistent with the previous studies.

V. CONCLUDING REMARKS

Quantum tunneling plays a key role in a plethora of phenomena beyond condensed-matter physics and applies to many different systems, including MOSFETs, resonant tunneling diodes (RTDs), electrical conduction in quantum dots, superconductivity, scanning tunneling microscopy, reaction kinetics, and biological processes.

A mechanism for quantum tunneling based on electron/hole annihilation and subsequent creation by the participation of real or virtual photons has been proposed, which can be generalized to any particle, given that every particle has an associated antiparticle. This mechanism circumvents the traditional and counterintuitive paradox of a particle with energy E lower than the barrier height V_0 being able to traverse the potential barrier. Furthermore, given that an energy gap could be treated in the manner of a potential barrier, as demonstrated by Zener,[19] this model can be applied to a number of other systems in which transitions would be mediated by virtual photons. Besides, the proposed mechanism adds up to the decades-old discussion on tunneling time and, in particular, is in accord with the Hartman effect.

Finally, the model here proposed can also be used to better understand resonant tunneling phenomena,[20] given that the participation of photons in these phenomena makes them somewhat similar to optical interference processes, such as those displayed by optical multilayers. In fact, a transfer matrix method can be used to solve both problems.[21]

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